

Abstract Book of the  
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*Mathematical Modeling of Self-Organizations in Medicine, Biology and Ecology: from micro to macro*

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## **KEYNOTE SPEAKERS' ABSTRACTS**

# Dynamics of neural networks

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The tools of dynamical systems theory are having an increasing impact on our understanding of patterns of neural activity. First I will introduce some of the more popular single neuron models and explain their behaviour in terms of bifurcation diagrams, phase-planes and phase-response curves. For limit cycle oscillators I will review the coupled oscillator approach that has provided a framework for understanding behaviour in neural networks with weak synaptic and gap junction coupling [1]. I will then show how results for strong coupling can be obtained by focusing on a specific class of spiking neural models, namely (non-smooth) planar integrate-and-fire models [2]. Next I will describe how to build tractable tissue level models that maintain a strong link with biophysical reality. These models typically take the form of nonlinear integro-differential equations. Their non-local nature has led to the development of a set of analytical and numerical tools for the study of waves, bumps and patterns, based around natural extensions of those used for local differential equation models [3,4]. Here I will present an overview of these techniques, and discuss the relevance of neural field models for describing the brain at the large scales necessary for interpreting EEG and MEG data. I will also present recent results on next generation neural field models obtained via a mean field reduction from networks of nonlinear integrate-and-fire neurons. I will show how this class of models can describe the phenomenon known as post-movement beta rebound, where a sharp increase in EEG/MEG power is seen in the beta frequency band following movement, typifying event-related synchronisation of brain activity [5].

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# Modeling swarms: from micro to macro

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## Abstract

A general class of mathematical structures (integro-differential equations) that can model self-organization at the so-called mesoscopic level is proposed. The equations are of kinetic type and the interactions have nonlinear nature and may be referred to as the mesoscopic scale of description [1]. The structures lead to interesting mathematical problems of blow-up of solutions [2, 3] that are directly related to swarming behavior. Both microscopic and macroscopic levels are also studied [4].

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# The emergence of spatial segregation patterns from animal movements and interactions

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Animal communities are often found segregated into different parts of space. This segregation sometimes occurs within a species, where small groups within a population each form a territory for their own exclusive use. But it can also occur between species, where competing species each live in a slightly different part of the landscape. Both these examples of spatial segregation occur without centralised control, emerging naturally from the movements and interactions of individual animals [1].

In this series of talks, we will examine the behavioural rules that govern these emergent phenomena. The first two talks will examine in detail a model of territorial scent-marking behaviour [2]. We will show how to analyse the model both (i) at non-equilibrium states using tools and concepts from statistical physics [2,3], and (ii) at equilibrium using both linear pattern-formation analysis and non-linear energy-functional techniques [4,5].

The final talk will look at how inter-species movement responses constitute a mechanism of spatially-segregated co-existence in populations where inter-species competition is greater than intra-species. This calls into question commonly-held assumptions about competitive exclusion [6].

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# Modelling cell migration

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It is nowadays understood that the interactions cells have with the fibrous environment they live in determine their behaviour. In fact, for instance, they play a fundamental role in cell motion, in tumour spread, in stem cell differentiation, and in tissue development.

Always starting from experimental facts, the lectures will describe several models that deal with different aspects characterizing these interactions. In particular, from the tutorial point of view, the topic will be used as a playground to briefly present several modelling frameworks, e.g., age-structured models, individual cell-based models, kinetic models, non local models, continuum mechanics, and mixture theory.

The models presented will be usually characterised by multiscale aspects because, for instance, the physical parameters determining cell motion and duplication are affected by the expression of proteins inside the cell and of receptors at its membrane. In turn, the behaviour of cellular aggregates and of tissues depends on the behaviour of single cells. For this reason, some upscaling tool will also be described.

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## **PARTICIPANTS' ABSTRACTS**

# Approximation in variation for the generalized sampling series

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## Abstract

The generalized sampling series are an important family of discrete operators not only from a mathematical point of view, but also because of their applications to Signal and Image Processing: indeed, this family of operators was introduced by P.L. Butzer in order to establish an approximate sampling formula, with the aim to reconstruct not-necessarily bandlimited signals (see, e.g., [4]). In this direction, we will present approximation results for the generalized sampling series defined as

$$(\bar{S}_w^m f)(t) := \sum_{k \in \mathbb{Z}} f\left(\frac{k}{w}\right) \bar{\chi}_m(wt - k), \quad t \in \mathbb{R}, \quad w > 0,$$

based on a family of kernel functions of averaged type, i.e.,  $\bar{\chi}_m(t) := \frac{1}{m} \int_{-\frac{m}{2}}^{\frac{m}{2}} \chi(t+v) dv$ ,  $t \in \mathbb{R}$ ,  $m \in \mathbb{N}$ , where  $\chi$  satisfies the usual assumptions on kernels and  $f$  is a function of bounded variation on  $\mathbb{R}$ . In particular, we first prove that such operators satisfy a variation detracting-type property, namely  $V[\bar{S}_w^m f] \leq \frac{1}{m} \|\chi\|_1 V[f]$ , for every  $w > 0$ ,  $m \in \mathbb{N}$ . Moreover, by means of a relation between the generalized sampling series and another important family of operators, i.e., the sampling Kantorovich series ([2, 5]), we are able to prove a characterization of the space of the absolutely continuous functions on  $\mathbb{R}$  in terms of convergence in variation, namely, for every fixed  $m \in \mathbb{N}$ ,  $V[\bar{S}_w^m f - f] \rightarrow 0$ ,  $w \rightarrow +\infty$ , if and only if  $f$  belongs to the space  $AC(\mathbb{R})$ .

We point out that the interest about approximation results in  $BV$ -spaces is due also to the important applications of such results in some problems of image reconstruction: indeed, the setting of  $BV$ -spaces is suitable in order to describe jumps of grey-levels of the image that correspond, from a mathematical point of view, to discontinuities ([3]).

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# The Kirchhoff-Plateau problem with elastic moving loop

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## Abstract

Often used for children's enjoyment, soap bubbles are physical examples of the rich mathematical problem of minimal surfaces: they assume the shape of the least possible area, containing a given volume. In my work I worked out the solution to the Kirchhoff-Plateau problem that is simply illustrated by soap-films that span flexible loops. This approach is a variant of the Plateau problem, a centuries-old mathematical problem. Plateau assumed that when you dip a rigid wire frame into a soap solution, the surface of the soap film formed on the frame has the minimum possible area, no matter the shape of the frame. In contrast to the Plateau's problem, in which a soap film spans a fixed frame, the Kirchhoff-Plateau problem concerns the equilibrium shapes of a system in which a flexible filament has the form of a closed loop spanned by a liquid film. In addition, we want to consider a loop formed by an elastic filament with non-vanishing cross-sectional thickness, and therefore it is essential to add an appropriate model to describe the behaviour of the loop in response to deformations. Indeed, the filament is modeled as a Kirchhoff rod, following the approach presented by Antman [1]. In this way the problem becomes "elasto-variational". In this framework I selected to study a specific, but more general, configuration of the loop. Starting from the problem solved by Giusteri et al. [3], I consider two thin elastic three-dimensional closed rods, where the first one is fixed, while the other one is free to move. The novelty of the problem consists also in the geometry: I require that the midline of each rod has to have linking number equal to one with the other one. Hence, we can say that they "are linked one to each other", namely they have to form what is called a single loop, like two linked rings. In order to do this, we have first, as usual, to impose the physical constraint of non-interpenetration of matter though allowing for points on the surface of the bounding loop to come into contact. Then, since the configuration is composed by two particular components, we have to require some additional conditions: by isotopy classes we first encode the knot type of the bounding loop and subsequently, since our rods form two tori, we require that the functions describing the midlines are two circumferences. I base my treatment of rod elasticity on Schuricht's approach [4]. Obviously, as for the bounding loop, we have to model the film and in particular the spanning surface. For the first one, we use a set with finite two-dimensional Hausdorff measure. Regarding the second condition, it turns out to be the most delicate point since we do not prescribe a priori the region where the liquid film touches the surface of the bounding loop. Since we are describing a three-dimensional situation, we adopt a DeLellis' result [2]: the spanning surface  $K$  has to be a  $C^1$ -spanning set of the bounding loop, i.e. it has to intersect all smooth embedding  $\gamma : S^1 \rightarrow \mathbb{R}^3 \setminus H$ , where  $H$  is the set occupying by the bounding loop. This approach is new because we do not have to prescribe the dimension of the filament or to require that it has to be fixed, since we want to consider an elastic rod. In the above described framework, the result I established in the thesis is the existence of an equilibrium shape that minimizes the total energy of the system: the elastic energy and the energy of the film. Furthermore, the addition of a moving component implies that its position accounts for possibly many experimental causes, some of which I experienced in the laboratory. In conclusion, this original research investigates a bit deeper the old Plateau's problem, which is very fascinating because its results are very closed to what happens in the physical world.

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# Seasonality in vaccination models with public intervention

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## Abstract

We present a game-theoretic investigation in the framework of behavioral epidemiology [2]. We assess the interplay between private vaccination choices and actions of the public health system (PHS) to favor vaccine propensity in SEIR-type diseases [1]. We explicitly focus on the role of seasonal fluctuations of the transmission rate. By means of analytical and analytical approximate methods, we investigate the global stability of the disease free equilibria. We show that in the general case the stability critically depends on the ‘shape’ of the periodically varying transmission rate. In other words, the knowledge of the average transmission rate (ATR) is not enough to make inferences on the stability of the elimination equilibria, due to the presence of the class of latent subjects. In particular, we obtain that the amplitude of the oscillations favors the possible elimination of the disease by the action of the PHS, through a threshold condition. Indeed, for a given average value of the transmission rate, in absence of oscillations as well as for moderate oscillations, there is no disease elimination. On the contrary, if the amplitude exceeds a threshold value, the elimination of the disease is induced. Numerical simulations support the theoretical predictions.

This research is a joint work with Alberto d’Onofrio (IPRI-Lyon).

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# Modeling and Self-Organization Dynamics of Aggregation Processes in Acoustically Levitated Disaccharides Solutions

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## Abstract

The present work deals with an analytical model formulated in order to characterize the drying process of acoustically levitated disaccharide aqueous solutions. Furthermore, the self-organization dynamics occurring in such disaccharide aqueous solution is addressed. In the latest decades, a growing attention has been focused to characterize the growth processes in disaccharides aqueous systems for both the academic and applicative point of views [1, 2]. The results obtained through the formulated analytical model are compared with experimental data collected, on aqueous solutions of trehalose and sucrose, by using an ultrasonic levitator and an infrared spectrometer [3]. It will be shown how the model predictions are in excellent agreement with the collected experimental data. Finally, the study reports an analysis of the resulting aggregated structures evidencing the different homogeneity degree for the investigated systems.

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# **Clibanarius erythropus and Phorcus turbinatus in intertidal ponds: evaluation of predator-prey interactions**

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## **Abstract**

The beach-rock is a peculiar sedimentary formation commonly appearing as a layered deposit inclined towards the sea. This formation is a hard substratum of natural origin, which represents a transition between the terrestrial and the marine environments. In some parts of the beach rock tidal ponds occur, and due to their jagged shape the water from the sea flows in, creating good environments inhabited by various and heterogeneous biocenosis with numerous taxa present [1]. Among the different taxa, Mollusca with *Phorcus turbinatus* and Crustacea with *Clibanarius erythropus* were the most represented. The predator-prey interaction between the abovementioned taxa has been studied for different environments and species, but no data are present about these species inhabiting the intertidal ponds. This study has been done using the non-invasive sampling technique of Visual Census in the intertidal ponds of Messina's beachrock (38°25'69"N, 15°61'24"E), from October 2016 to July 2017. One of the first feature noted during the visual census survey was that, in numerous cases, *Clibanarius erythropus* was found inhabiting the shell of *Phorcus turbinatus*. This characteristic clearly indicates that *Phorcus turbinatus* specimens were preys of *Clibanarius erythropus* which, as all the crustaceans, do not built by themselves the shell and need to steal one (from molluscans) to protect their body. It has been also observed that such populations may exhibit periodic fluctuations. A goal of this study is to obtain a mathematical model capable of describing most of the observed predator-prey dynamics.

## **References**

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# Optimal transport in living systems: physiological network and Kleiber law

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## Abstract

It has been pointed out that transportation of metabolites in living systems obeys a variational/optimization principle [1] which leads to a simple geometrical derivation of the Kleiber's law [3] according to which the basal metabolic rate of a living being scales like its mass to the  $3/4$  power law over a range of several orders of magnitude [4]. Here we present a continuum formulation by mapping the problem in the optimal transportation principle as first formulated by Monge and subsequently relaxed in a weaker format by Kantorovich. This new principle, which appears precisely the continuous version of the variational principle in [1], captures the geodesic character of physiological transportation network of metabolites in living systems similar to the Fermat principle of the geometrical optics. The present talk is based on a research in collaboration with Amos Maritan and Jayanth Banavar [2].

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# Multiscale mathematical and physical model for the study of nucleation processes in meteorology

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## Abstract

The development of numerical models for meteorological analysis [1-5] is often based on nonlinear equations systems describing complex fluids, having more components and for which generalized Navier - Stokes equations are used. In several real cases it is necessary to use numerical calculation approaches in which the equations are discretized. The aim of this contribution is working out results regarding the optimization of the performance of a mathematical - physical model, formulated for meteorology forecasts in limited and high resolution area, and obtained by interdisciplinary studies. In particular, in order to test the validity of the presented model [1], different microphysical parameterizations have been analyzed, every one taking into account different hydrometeor nucleation processes.

## References

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# A quest towards the modeling of human behavior in economics and social sciences

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## Abstract

Recent radical changes in the modeling of social and economical sciences, leading to interesting cultural developments in the social sciences, push the search for mathematical tools able to deal with the complexity of behavioral features of these systems in the research activity, see [2, 3, 4, 1]. Namely, a constructive interplay between mathematics and the socio-economics analysis can be properly developed as far as mathematical tools help to take into account the complexity features of behavioral systems. A successful interaction can lead to exploratory models which can depict qualitatively the behavior of social-economical systems for by exploring their asymptotic behaviors corresponding to different choice of the parameters.

The KTAP modeling approach has been applied in recent years to the study of a variety of social and political systems, although applications to economics are still confined at a primitive stage, still waiting for possible development. This talk accounts for the role of human behaviors on the overall dynamics and aims at showing how these “social behaviors” and “political choices” can have an important influence on the dynamics of our society.

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# Non-coding RNAome of oxidative stressed and normal RPE cells suggests unknown regulative aspects of Retinitis pigmentosa etiopathogenesis

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## Abstract

The discovery of thousands of non-coding RNAs, both small and long, has revolutionized molecular biology. These non-coding RNAs have been implicated in several biological processes and diseases ([1]). In order to clarify oxidative stress role on Retinitis pigmentosa (RP), a very heterogeneous and inherited ocular disorder group characterized by progressive retinal degeneration, we realized a comparative transcriptome analysis of human retinal pigment epithelium cells, treated with oxLDL and untreated, in four time points (1h, 2h, 4h, 6h). An ion Proton<sup>TM</sup> sequencer was used for thrice repeated experiment. Data analysis foresaw a complex pipeline, starting from Burrows-Wheeler (BWA), STAR and TopHat2/TopHat-Fusion alignments comparisons, followed by transcriptomes assembly and expression quantification by Cufflinks. Then we compared previously obtained data to several known transcript databases (UCSC, Ensembl, iGenome, GENCODE, DASHR, LNCipedia, miRBase), in order to filter out ncRNAs. Finally, we completed the computational analysis roadmap with several python - based tools specific for Circular RNAs (CIRCexplorer, UROBORUS, CIRI and KNIFE), one Perl - based script for Long non-coding RNAs (FEELnc), and the Small RNA Analysis tool included in Qiagen CLC Genomics Workbench software. All data were validated by Bonferroni - corrected EDGE statistical analysis. Finally, all detected ncRNAs underwent pathway analysis by Cytoscape software. We found, in treated and untreated samples, changing their own expression throughout all considered time points, 53,422 lncRNAs, the most of them divided in 986 Long Intergenic non-coding RNAs (lincRNAs), 12 Intronic lncRNAs, 256 Sense lncRNAs, 1472 Antisense lncRNAs, 361 Circular Intronic RNAs (ciRNAs) and 1581 Exonic Circular RNAs (circRNAs). Furthermore, 708 Small non-coding RNAs (sncRNAs) were detected, divided in 157 Micro RNAs (miRNAs), 248 Small Nucleolar RNAs (snoRNAs), 4 Small Nuclear RNAs (snRNAs), 133 Spliceosomal RNAs and 166 Piwi-interacting RNAs (piRNAs). All found ncRNAs showed several common pathways, in which they could interact each other and/or directly regulate gene expression. The most relevant of them are "Nitrogen, heterocyclic and aromatic compounds metabolic processes", "Vesicle trafficking", "Ubiquitin proteasome pathway", "Spliceosomal assembly", "Fatty acids metabolism" and "Transmembrane transport". These data suggest that ncRNAs could play a relevant role in RP etiopathogenesis, regulating pathways directly related to already known causative genes or not yet associated with the considered disease.

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# Patterns in "dirt" melting snow

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## **Abstract**

Alternating layers of snow and volcanic ash, as can be found at times on Etna, create a peculiar pattern of small ash heaps when the snow melts under the sun. This work tries to model this phenomenon.

# Modeling sorption of trace-elements in multispecies biofilms

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## Abstract

The presentation will concern a mathematical model for micro-elements sorption in multispecies biofilms, based on a continuum approach and mass conservation principles. Diffusion of contaminants within the biofilm is described using a diffusion-reaction equation. Binding sites formation and occupation are modeled by two systems of hyperbolic partial differential equations mutually connected through the two growth rate terms. The model is completed with a system of hyperbolic equations governing the microbial species growth within the biofilm, a system of parabolic equations for substrates diffusion and reaction and a nonlinear ordinary differential equation describing the free boundary evolution. The Mathematical Modelling of three real special cases will be presented. The first describes the dynamics of a free sorbent component diffusing and reacting in a multispecies biofilm. In the second illustrative case, the fate of two different trace-elements is modelled. In the third case the microbial growth is directly related to the contaminant concentration.

# Oscillatory patterns in reaction-diffusion systems with cross-diffusion

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## Abstract

Complex spatio-temporal dynamics in reaction-diffusion systems is generally ascribed to the interaction between the Turing and Hopf instabilities, which can occur either through a codimension-two Turing-Hopf bifurcation or due to different competing bifurcations of multiple steady states. In particular, the observed dynamics in the proximity of a codimension-two Turing-Hopf bifurcation point can be classified in two different groups: the first includes the dynamics resulting from the interplay between a Turing mode and a Hopf mode, the second group consists of dynamical behaviors originated by subharmonic instabilities of the Turing and the Hopf modes. In this talk we shall investigate the complex spatio-temporal dynamics arising in two different reaction-diffusion systems, which is due to the two above described mechanisms. At first we analyze the activator-depleted substrate Schnakenberg system with linear cross-diffusion introduced in [1], which support both Turing and Hopf instabilities. We investigate the local dynamics in the proximity of the codimension-two Turing-Hopf point and unfold the codimension-2 Turing-Hopf bifurcation through the adoption of normal forms. The analytical and numerical studies performed show that the interplay of stationary inhomogeneous patterns with homogeneous oscillatory patterns leads to the emergence of oscillating structures characterized by the presence of one wavenumber  $k_c$  and one frequency  $\omega_c$ , with all peaks oscillating synchronously (in-phase oscillations). In addition, we illustrate how the dynamics of the ODEs associated with the normal forms can be reliably used to predict the behavior of the solutions close to the bifurcation point. Moreover, our investigations away from the bifurcation reveal multistability between oscillatory and stationary pattern and the occurrence of chaotic dynamics. Secondly, we will consider a predator-prey reaction-diffusion system with a Lotka-Volterra reaction kinetics and a nonlinear cross-diffusion term in the predator equation, which does not exhibit any Hopf or wave instability [2]. Therefore, on the basis of the linear analysis one should only expect stationary pattern. The presence of the nonlinear cross-diffusion term is able to induce a subharmonic destabilization of the fundamental subcritical Turing (subT) mode, giving rise to a mixed mode solution oscillating out-of-phase. We show that this subharmonic solution can undergo a sequence of secondary instabilities, generating a transition towards chaotic dynamics. We finally investigate the formation of subT-modes on 2D domains: when the fundamental mode describes a square pattern, subharmonic resonances originate oscillating square patterns. In the case of subcritical Turing hexagons solutions, the internal interactions with a subharmonic mode are able to generate the so-called "twinkling eye" pattern.

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# Mathematical Model of Acute inflammation and its analysis of instabilities induced by chemotaxis

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Inflammation is the body's response to outside threats like stress, infection, pathogens or damaged cells and its aim is to promote tissue repair and healing. It is characterized by the action of both pro- and anti-inflammatory agents that work together to ensure a quick restoration of tissue health. However, a dis-regulation of the inflammatory response can give rise to chronic inflammation and lead to a wide range of disease, like cancer, atherosclerosis and asthma. Understanding the precise role of every inflammatory agent is the key to design a treatment for minimizing tissue damage and chronic disease. Recently, several mathematical models were proposed to test biological hypothesis and improve the knowledge of the inflammatory process.

Penner et al.(2012) [1] have proposed a spatio-temporal model describing the dynamics of inflammation consisting of a fixed population of immune cells, such as macrophages, and two type of signalling molecules: a chemokine, which is the chemoattractant for the immune cells, and an anti-inflammatory cytokine, which acts as inhibitor. We shall generalize the model proposed by Penner et al.(2012) [1] including a kinetics for the macrophages, in order to take into account the effects of immune cells recruitment, that is crucial during inflammatory processes.

The proposed model consists of a Reaction-Diffusion-Chemotaxis system, which supports the formation of spatio-temporal patterns. We shall therefore present a stability analysis close to the homogeneous steady state and some theorems giving the necessary and sufficient conditions for the occurrence of both Turing and wave instability.

We shall also investigate the far from the equilibrium dynamics and, therefore, the secondary instabilities that arise when the control parameter is increased beyond the primary bifurcation.

Finally, we shall analyze the stability and instability arising in our chemotaxis model and find the corresponding conditions for different chemotactic strengths by using energy estimates, spectral analysis and bootstrap arguments.

All the theoretical predictions of the analysis have been numerically tested adopting realistic values of the parameters derived from experimental data taken from the literature.

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# Morphogenesis in metal growth by electrodeposition

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## Abstract

Alloy electrodeposition (ECD) processes have been experimentally shown to exhibit electrokinetic instability that can lead to compositional heterogeneity in the electrodeposit bulk. We propose a reaction-diffusion model - coupling surface morphology and surface composition - as a mean of understanding the formation of morphological patterns found in the electrodeposition processes. We investigate the related spatiotemporal dynamics from both the analytical and numerical points of view, revealing interesting classes of morphogenetic scenarios in good qualitative accordance with experiments. We study Turing and Hopf instabilities and analyse the codimension-2 Turing-Hopf bifurcation. As a result, we detect and characterize classical Turing patterns as well as oscillating Turing patterns. In the Hopf region, we prove the existence of spiral wave behaviour and observe transition to spatiotemporal chaos via spirals break up. The role of cross-diffusion in pattern formation and selection is also discussed. The obtained results can have direct industrial applications since they allow a control strategy of ECD morphology based on manipulations of the surface chemistry. In this direction we have shown that an applied time dependent forcing term implies smoothing effects on the morphochemical patterns. This suggests that "green" solutions could be provided for electroplating technologies that are traditionally based on the use of non-sustainable chemistries.

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# Chemotactic models of plaque formation and remyelination in Multiple Sclerosis

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## Abstract

Multiple sclerosis (MS) is an immune-mediated inflammatory disease that attacks myelinated axons in the central nervous system, destroying the myelin and the axon. Despite of the significant advances in the understanding of the molecular and cellular processes involved in the numerous mechanisms of demyelination and neuronal injury, it is a major challenge in MS research to define which of these mechanisms are relevant within MS lesions and what is their relative contribution at different stages of the disease. In this talk we shall review a recently derived mathematical model describing the primary mechanisms that drive the early stages of the disease, characterized by activated local microglia, with the recruitment of a systemically activated immune response and by oligodendrocytes apoptosis [1]. The model consists of a reaction-diffusion-chemotaxis system which, adopting a combination of analytical and numerical approaches, we have proved to support the formation of different demyelinating patterns. Using biological parameters from the literature, we have in fact observed the appearance of localized areas of apoptotic oligodendrocytes, which closely fit existing MRI findings on the active MS lesion during acute relapses; concentric rings, typical of Balòs sclerosis; small clusters of activated microglia in absence of oligodendrocytes apoptosis, observed in the pathology of preactive lesions. We shall also discuss some further extensions of the model: in particular we shall take into account the endogenous regenerative efforts, driven by the immune response and associated with acute stages of inflammation, which promote remyelination, a robust regenerative response, well documented in the early phases of the disease, but which often fails in chronic lesions [2]. We shall finally propose a modified version of the above model to test a different mechanism in the aetiopathogenesis of MS proposed by Barnett and Prineas [3]. In fact, examination of the early evolving lesions from patients with relapsing-remitting disease suggests that a previously unrecognized prephagocytic phase precedes classically defined lesion activity; and that the primary injury to the oligodendrocytes underpins the targeting of these structures by macrophages.

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# A SEIR mathematical model of vector-borne disease epidemics and its simulation

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## Abstract

Infectious vector-borne diseases are kept in nature from a primary cycle (also called "endemic cycle") of transmission vector-reservoir-vector :adult vectors are infected with viremi-reservoir. Once swallowed, the pathogen is in the vector organism and is then transmitted to the vertebrate host. The secondary cycle (also called "epidemic cycle") manifests itself when accidental hosts (dilution hosts), enter the transmission cycle and are affected by the infection. A SEIR mathematical model of vector-borne infective disease is presented, taking account the local interactions between reservoir and vectors, as well as the transmission from vectors to dilution hosts. In this model vectors possess the ability to keep the virus within their own population through vertical transmission. We show the existence and stability of disease-free equilibrium point, define the basic reproductive number  $R_0$  and the reactivity index  $\varepsilon_0$ , and discuss the existence of endemic equilibria. Finally, we quantify the effects of vertical transmission on long-term persistence of vector-borne disease using numerical simulations.

# **The invasion problem in biofilm reactors: qualitative analysis and numerical applications**

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## **Abstract**

The work presents the qualitative analysis of the free boundary value problem related to the invasion process in biofilm reactors. In the framework of continuum approach to mathematical modelling of biofilm growth, the problem consists of a system of nonlinear hyperbolic partial differential equations governing the microbial species growth and a system of semi-linear elliptic partial differential equations describing the substrate diffusive trends. The model is completed with a system of elliptic partial differential equations governing the diffusion and reaction of planktonic cells, which are able to switch their mode of growth from planktonic to sessile when specific environmental conditions are found. Two systems of nonlinear differential equations for the substrate and planktonic cells mass balance within the bulk liquid are also considered. The free boundary evolution is governed by a differential equation which accounts for the displacement velocity due to the microbial biomass and the detachment flux as well. The main mathematical methodology used is the method of characteristics, which has enabled us to convert the original differential problem into integral equations. The fixed point theorem has been applied for the uniqueness and existence results. The work is completed with some numerical applications related to a real engineering/biological case which examines the invasion of specific microbial species in a constituted biofilm, inhabiting the deammonification units of the wastewater treatment plants. For all the cases analyzed real data from existing literature have been used to feed numerical simulations, which produce results in agreement with experimental findings.

# Operatorial Models in Ecology

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## Abstract

Since 2006, it has become clear that raising and lowering operators of quantum mechanics can be successfully used for the mathematical description of some macroscopic systems in several contexts, such as social life and decision-making processes, political systems, population and crowd dynamics, ecological processes.

The models we consider are built by using *fermionic* annihilation and creation operators for each compartment of the model, and their associated number operators are used to define local densities of the observables. The dynamics is assumed to be ruled by a time-independent self-adjoint Hamiltonian operator. In the Heisenberg picture, the time evolution of an observable  $X$  of the macroscopic system  $\mathcal{S}$  we are considering is given by  $X(t) = \exp(iHt)X \exp(-iHt)$ , where  $H$  is the Hamiltonian of  $\mathcal{S}$ , and the mean values of the observable  $X$  are linked to real valued functions phenomenologically associated to some macroscopic quantities.

For quadratic Hamiltonians the dynamics that can be deduced is at most quasiperiodic; allowing some of the parameters involved in the Hamiltonian to be complex, it is possible to describe also dissipative processes. Moreover, in order to obtain non-trivial time evolutions, we assume the dynamics to be driven, besides the Hamiltonian, by the periodic action of certain *rules*  $\rho$  on the system able to modify the values of some of the parameters on the basis of the state variations of the system. The latter approach, recently introduced, is called  $(H, \rho)$ -induced dynamics.

Some applications for the description of closed ecological systems and for the dynamics of stressed bacterial colonies, as well as to model desertification processes, are presented.

# Analysis of solutions for a cerebrospinal fluid model

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## Abstract

In this talk we will provide a rigorous mathematical study for the dynamics of the cerebrospinal fluid (CSF). Although the mathematical modeling of the intracranial dynamics has gained interest and many researchers have developed and analyzed models of different complexity ([4]), in the literature there are no rigorous results regarding the analysis of solutions to the system of equations which describe the different mechanisms in the intracranial pattern.

We start from a model developed by Linninger et al. ([2], [3]) which is given by the following system of equations

$$\begin{cases} \partial_t \eta(t, z) + \partial_t a(t) + u(t, z) - \tilde{Q}_p = 0, \\ \alpha \partial_{tt} \eta(t, z) + \tilde{k} \partial_t \eta(t, z) + \kappa \eta(t, z) - AP(t, z) + A\tilde{P} = 0, \\ \rho \partial_t u(t, z) + \rho u(t, z) \partial_z u(t, z) + \partial_z P(t, z) + \beta u(t, z) = 0, \end{cases} \quad (1)$$

where  $u$  is the CSF velocity flux,  $\eta$  is the tissue displacement and  $P$  is the pressure,  $a(t)$  is the forcing function and the other parameters are physical constants and physiological data. In this first approach we neglect the variation of the cross section  $A$  due to the choroid plexus expansion. The equations describe the velocity of the CSF flux, the tissue displacement and the evolution of the intracranial pressure. In particular the equation for the velocity is of Burger type.

We will show, by means of an iterative process, the existence and uniqueness of a local solution in time. Then, we investigate under which conditions it is possible to obtain global existence. Moreover, the last part of the talk will be devoted to present numerical simulations for the analyzed cerebrospinal model. In particular, in order to assess the reliability of the stated theoretical results, we will carry out numerical simulations in two different cases: when initial data satisfy the conditions for the global existence of solutions or when they violate them, see [1].

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# Oscillating localized solutions and homoclinic snaking in the FitzHugh-Nagumo model

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The FitzHugh-Nagumo (FN) model has been introduced to model the conduction of electrical impulses along axons[1]: adopting simple non linearities, the kinetics describes the main features of an excitable media and, depending on the parameters values exhibits bistability of the steady states. The phenomenon of wave propagation supported by this model has been largely studied also in the case when the kinetics is coupled with self and cross diffusion terms [2]. In this talk we are interested in the study of pattern formation in the FN model with full linear diffusion matrix. We shall therefore consider the following adimensionalised reaction-diffusion system:

$$\begin{aligned}\frac{\partial u}{\partial t} &= \Gamma[-u^3 + u - v] + \nabla^2 u + d_v \nabla^2 v \\ \frac{\partial v}{\partial t} &= \Gamma[\beta(u - \alpha v)] + d_u \nabla^2 u + d \nabla^2 v\end{aligned}$$

where the parameters are non negative. Through a linear instability analysis we shall determine the conditions for the onset of Turing instability[3]. We shall show how the presence of cross diffusion terms allows for the existence of a double threshold for the Turing instability: this implies that, in contrast with the requirement of classical Turing condition, for suitable values of the parameters, stationary patterns can appear even for small values of  $d$ . We shall also investigate the system dynamics far from equilibrium, considering the case when a subcritical transition occurs: we determine the location of the Maxwell point, in the neighborhood of which localized structures can emerge. We numerically compute the corresponding bifurcation diagrams showing the occurrence of homoclinic snaking[5] and the presence of different branches which correspond to qualitatively different solutions. We finally investigate the possibility of oscillatory in time solutions: since the kinetics of the system exhibits Hopf bifurcation, through a multiple scales analysis, we compute the normal form of the codimension two Turing-Hopf bifurcation deriving the corresponding amplitude equations[4]. We finally present some numerical simulations proving, close to the codimension two Turing-Hopf bifurcation point, the presence of oscillating localized solutions, pinning solutions and chaotic behaviour.

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# Approximation problems for the diagnosis of vascular diseases

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## **Abstract**

Convergence and order of approximation results for suitable families of discrete operators, are analyzed and discussed in order to process digital biomedical images and to detect the vessel lumen in CT-images of the aorta artery without the use of contrast medium. The study can be useful to diagnose aneurysm pathologies

# Design and steering of a magneto-elastic micro-swimmer inspired by the motility of sperm cells

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## Abstract

Controlling artificial devices that mimic the motion of real microorganisms, is attracting increasing interest, both from the mathematical point of view and applications. A model for a magnetically driven slender micro-swimmer, mimicking a sperm cell is presented, supported by a feasibility study for its realization. Using the well known Resistive Force Theory (RTF) approach to describe the hydrodynamic forces, the micro-swimmer can be described by a driftless affine control system where the control is an external magnetic field. Moreover we discuss through numerical simulations how to realize different kind of paths.